

# OLIMPIADA NAȚIONALĂ DE MATEMATICĂ

## HUNEDOARA

### ETAPA LOCALĂ

#### CLASA a V-a

1. a)  $A = 10 \cdot (3+3^2+3^3+\dots+3^{2018}+10)$

$$A+B = 10 \cdot (3+3^2+3^3+\dots+3^{2018}+10) + (3+3^2+3^3+\dots+3^{2018}+10) \quad \mathbf{1p}$$

$$A+B = (3+3^2+3^3+\dots+3^{2018}) \cdot (10+3+3^2+3^3+\dots+3^{2018}) = (3+3^2+3^3+\dots+3^{2018}+10)^2 \quad \mathbf{2p}$$

b)  $A = 2 \cdot 5 \cdot (3+3^2+3^3+\dots+3^{2018}+1+9) = 5 \cdot [2 \cdot (1+3+3^2+3^3+\dots+3^{2018})+18]$

$$A = 5 \cdot (3^{2019} - 1 + 18) = 5 \cdot (3^{2019} + 17) \quad \mathbf{2p}$$

$$5 \cdot 3^{2019} + 5 \cdot 17 < 5 \cdot 3^{2019} + 4 \cdot 3^{2019} = 9 \cdot 3^{2019} = 3^{2021} \quad A < 3^{2021} \quad \mathbf{2p}$$

2.  $M = 4^n \cdot 4 \cdot 9^n \cdot 9^2 - 6^{2n} \cdot 6 \cdot 9 - 2^{2n} \cdot 9^n \cdot 9 \quad \mathbf{2p}$

$$M = 36^n \cdot 4 \cdot 81 - 36^n \cdot 6 \cdot 9 - 36^n \cdot 9 \quad \mathbf{3p}$$

$$M = 36^n \cdot 9 \cdot (36 - 6 - 1) = 36^n \cdot 9 \cdot 29 = 36^n \cdot 261 : 261, \text{ pentru orice } n \text{ număr natural.} \quad \mathbf{2p}$$

3.  $n = 2003 \cdot c + r, \quad r < 2003 \quad \mathbf{1p}$

$$r \in \{0, 1, 2, 3, \dots, 2002\}$$

$$n = 2003 \cdot r + r \Rightarrow n = 2004 \cdot r \quad \mathbf{1p}$$

$$S = 2004 \cdot 0 + 2004 \cdot 1 + 2004 \cdot 2 + 2004 \cdot 3 + \dots + 2004 \cdot 2002 \quad \mathbf{1p}$$

$$S = 2004 \cdot (1+2+3+\dots+2002) \quad \mathbf{1p}$$

$$S = 2004 \cdot [(2002 \cdot 2003) : 2] \quad \mathbf{2p}$$

$$2S = 2002 \cdot 2003 \cdot 2004 \quad \mathbf{1p}$$

4.  $\overline{xy} \cdot (2x+y-z) = 75$

Avem cazurile:

a)  $\overline{xy} = 75 \Rightarrow x=7, y=5, z=18 \text{ (fals)} \quad \mathbf{3p}$   
 $2x+y-z = 1$

b)  $\overline{xy} = 25 \Rightarrow x=2, y=5, z=6 \text{ (adevărat)} \quad \mathbf{2p}$   
 $2x+y-z = 3$

c)  $\overline{xy} = 15 \Rightarrow x=1, y=5, z=2 \text{ (adevărat)} \quad \mathbf{2p}$   
 $2x+y-z = 5$

**Notă:** Orice altă rezolvare corectă se va puncta corespunzător.